**Optimality**

Let's look at some pair-division of our array and pick two pairs made of the numbers a, b, c, d (in some pair-division) – assuming . We will prove that switching them so that they will be constructed from the (a, d), (b, c) can only improve our total result (more specifically – not to increase the sum difference between the pairs). Therefore, we can change any solution to our solution by steps that cannot increase the sum-diff, and therefore it is the optimal solution.

We have, without loss of, generality, only 3 possible cases:

1. Original pairs are (a, d), (b, c):  
   we are already at the desired result, no harm indeed.
2. Original pairs are (a, b), (c, d):  
   current diff is , new diff is or (depends on the absolute value). In the first case, the diff is , and in the second the diff is , anyway we cannot increase the sum-diff.
3. Original pairs are (a, c), (b, d):  
   current diff is , new diff is or (depends on the absolute value). In the first case, the diff is , and in the second the diff is , anyway we cannot increase the sum-diff.